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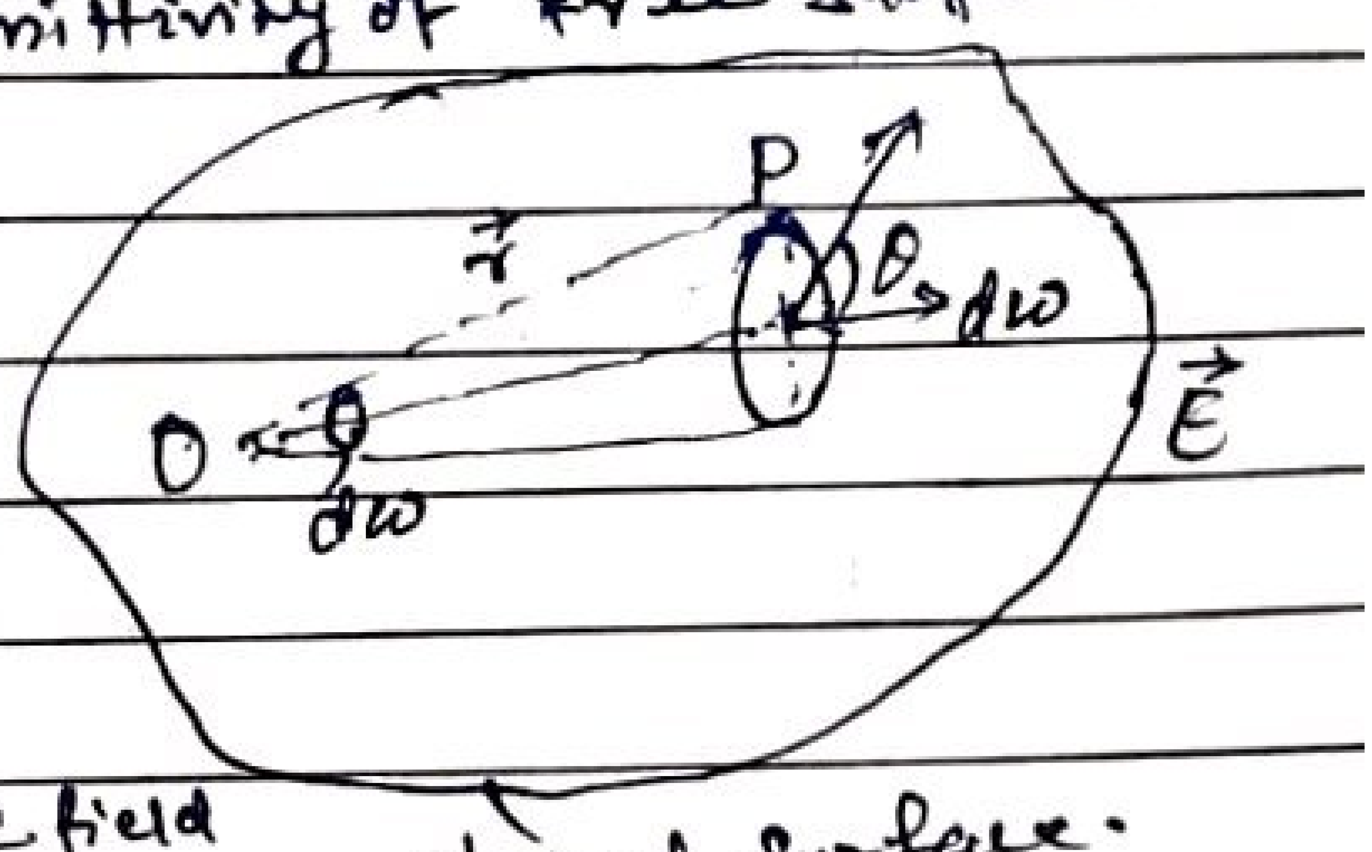
Gauss law and its applications

The total electric displacement or electric flux through any closed surface surrounding charges is equal to the net positive charge enclosed by that surface.

i.e $\phi_E = \oint_S \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$

Here ϵ_0 denotes the absolute permittivity of free surface.



Proof:– Let there is a close surface of any shape which is passing through from point P. this surface is known as Gaussian surface.

According to coulomb law, the electric field at a point is given by $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

Suppose $d\vec{A}$ is small area element at point P.

By defined of electric flux, we have.

$$\phi_E = \oint \vec{E}\cdot d\vec{A} = \oint |\vec{E}|\cdot|d\vec{A}|\cos\theta$$

where θ is angle between $\vec{E}$ and $d\vec{A}$

$$\phi_E = \oint\left\{\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\right\}dA\cos\theta = \frac{1}{4\pi\epsilon_0}Q\cdot\oint\frac{dA\cos\theta}{r^2}$$

or, $\phi_E = \frac{1}{4\pi\epsilon_0}Q\cdot\oint d\omega$ $\quad\left(d\omega = \frac{dA\cos\theta}{r^2}\right)$ Definition of solid angle.

We know that the surface integration of $d\omega$ or total solid angle by close surface at inside the close surface is 4π.

Thus $\oint d\omega = 4\pi$

$$\therefore \phi_E = \frac{1}{4\pi\epsilon_0}Q\cdot 4\pi = \frac{Q}{\epsilon_0}$$

i.e $\phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$

Hence Gauss theorem is proved.

Special cases:

Case I - If no charge is enclosed

Then, $\oint \vec{E}\cdot d\vec{A} = 0$ or, Electric flux $\phi_E = 0$

Case-II If the surface enclosed discrete distribution of charges in medium of dielectric const. ϵ_r,

Then, $\phi_E = \int \vec{E}\cdot d\vec{A} = \frac{1}{\epsilon_0\epsilon_r}\sum_{i=1}^{n} q_i$

Case-III If the charge is uniformly distributed, in free space, inside the surface.

Then $\phi_E = \frac{1}{\epsilon_0}\int \rho\, dv$. ~~and~~ dv is the small volume element.

where ρ is the volume charge density